Bounded-uncertainty estimation for correlated signal and noise

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Abstract

In this paper we present a class of bounded-uncertainty estimators as the solution of a classic estimation problem involving unknown statistics. The estimators are derived under the non-typical assumption of correlated signal and noise. The bounded-uncertainty framework gives an additional degree of freedom for estimator design that can benefit its performance. It also provides an indirect way of verifying hypotheses regarding unknown variable statistics in a particular application domain by examining the behavior of the estimators as a function of the bound(s). If the unknown statistics are within a lower bound than the worst-case limit assumed by a classic minimax estimator, the quality of the estimation is increased by this new approach.

I. Introduction

The problem of estimating a signal in additive i.i.d. noise in the context of image denoising has continued to receive attention in the literature (e.g. [1], [2]). Classical solutions such as Wiener filtering are available for the core estimation problem, depending on the assumptions on signal and noise statistics. In general the statistics of some intervening variables may not be known. Then, an estimator is sought whose performance is optimized for the worst-case scenario on the unknown statistics. This approach yields the widely-used minimax MSE estimator [3].

The type of image denoising filters we consider have at their core a signal estimator. The denoising operates in an overcomplete transform domain, with the advantage that multiple estimates of an image sample can be filtered to obtain a better final estimate. These filters can perform a simple averaging, or a weighted-averaging of the estimates [4]. An important characteristic of signal and quantization noise in image and video coding is that they are correlated. In most practical cases, it is difficult to estimate this correlation. This leaves as an option the use of minimax estimation. It is well-known that the performance of minimax estimators is often unsatisfactory in that they are overly conservative. This feature results from their intrinsic rigid optimization for the worst-possible choice of unknowns.

In this paper we propose an approach for designing a more general class of estimators that achieve a better balance between performance and robustness conditions. The estimation problem is formulated and solved under the important assumption of correlation between signal and quantization noise. Rather than follow the classic minimax formulation in this context, we describe a more general approach to the determination of an optimal linear estimator. This approach is based on the placement of a bound in the feasible interval of the signal-noise correlation, and the derivation of the estimator as a function of the bound and of estimates of the signal and noise power. Various formulations can be used for deriving this class of estimators, ranging from placing a bound in the feasible interval of the unknown variable, to endowing it with a distribution function. An average risk formulation is also considered, where the estimation criterion is designed to express a degree of confidence in a particular assumption about the unknowns, counter-balanced by a term that handles violations of that assertion.

II. Bounded-uncertainty estimators and quantization noise filtering

Let us consider the signal model:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \tag{1}$$

with zero mean signal and noise, the signal $\mathbf{x}$ has covariance C_x, the noise $\mathbf{w}$ has covariance C_w, C_{xw} denotes the signal and noise correlation matrix, and $\mathbf{y}$ is the noisy observed signal. Let $\mathbf{H} = \mathbf{I}$. Assuming C_x, C_w known, the objective is to find a linear estimator $\hat{\mathbf{G}}$, which applied to $\mathbf{y}$ will produce a best estimate $\hat{\mathbf{x}}$ of $\mathbf{x}$, in a MMSE sense. Thus,

$$\hat{\mathbf{x}} = \hat{\mathbf{G}}\mathbf{y}. \tag{2}$$

Since in the context of image filtering in a transform domain we are going to denoise separately each transform coefficient, let us consider the scalar version of Eq. (1) and denote by σ_x^2, σ_w^2, σ_{xw} the corresponding statistics. It is assumed that σ_x^2,

σ_w^2 are known (estimated). The estimator matrix G reduces to a scalar estimator g. The cross-correlation σ_{xw} is unknown, but from the positive-definite requirement on the system matrix of (1) it results that $|\sigma_{xw}| < \sigma_x\sigma_w$. The minimax estimator is [4]:

$$\begin{array}{ll} \textit{if} & \sigma_x^2 \leq \sigma_w^2 \\ & \hat{g} = 0. \\ \textit{else} & \\ & \hat{g} = 1. \end{array} \tag{3}$$

To formulate the class of bounded-uncertainty estimators (BU-EST), let us place a bound $b > 0$ in the feasible interval of σ_{xw}, i.e. $|\sigma_{xw}| < b \leq \sigma_x\sigma_w$, and determine optimal estimators that are a function of the bound b, as well as of signal and noise power. In a first formulation, the estimator $\hat{g}$ is obtained as follows:

$$\hat{g} = \arg\min_{g} \max_{\substack{\sigma_{\mathrm{xw}}, \\ |\sigma_{\mathrm{xw}}|<\mathrm{b}}} MSE(g, \sigma_{xw}). \tag{4}$$

The estimator that results has the following form:

$$\begin{array}{lll} \textit{if} & \sigma_x^2 \leq b & \\ & \hat{g} = 0. & \\ \textit{else} & & \\ & \textit{if } \sigma_w^2 \leq b & \\ & & \hat{g} = 1. \\ & \textit{else} & \\ & & \hat{g} = \frac{\sigma_x^2 - b}{\sigma_x^2 + \sigma_w^2 - 2b}. \end{array} \tag{5}$$

As shown by the structure of the estimator, the case of interest is when $b < \min\{\sigma_x^2, \sigma_w^2\}$, where the estimator performs a non-trivial scaling of the noisy signal.

In terms of practical implementation, the condition $b < \min\{\sigma_x^2, \sigma_w^2\}$ can be obtained by setting $b = \alpha\sigma_w^2$, $0 < \alpha < 1$. The conditions in (5) change accordingly, and the transfer function is shown in Fig. 1. The abrupt transition made by a minimax estimator is softened by the BU-EST according to the rational quadratic function in Eq. (5). Since in this implementation the

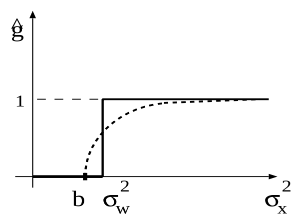

Fig. 1. Transfer functions.

estimator only tends asymptotically to 1 as σ_x^2 increases, an additional threshold can be set such that when

$$(\sigma_x^2 - b) = K(\sigma_w^2 - b), \tag{6}$$

where $K > 1$, the estimator is set to 1 ($\hat{g}$=1). A further approximation can be made to increase computational efficiency (for an overcomplete operation) by replacing the rational quadratic functional requiring a division by a linear approximation.

In a second formulation of a BU-EST, let us define a distribution function $F(b) = P\{\sigma_{xw} \leq b\}$. Then, we obtain the estimator as follows:

$$\hat{g} = \arg\min_{g} \{ \max_{\substack{\sigma_{\mathrm{xw}}, \\ |\sigma_{\mathrm{xw}}|<\mathrm{b}}} F(b) MSE(g, \sigma_{xw}) + \max_{\substack{\sigma_{xw}, \\ b \leq |\sigma_{xw}|}} [1 - F(b)] MSE(g, \sigma_{xw}) \}. \tag{7}$$

Since $F(b)$ is monotonically-increasing, as b increases so does the first term in Eq. (7) relative to the second. The choice of $F(.)$ controls the characteristics of this dependency. This implies that as the interval bounded by b is expanded, the r.v. σ_{xw} is more likely to be inside that interval. The BU-EST that results is similar in form to the previous case, but with a new variable

γ intervening:

$$\begin{array}{lll} \textit{if} & \sigma_x^2 \leq \gamma & \\ & \hat{g} = 0. & \\ \textit{else} & & \\ & \textit{if } \sigma_w^2 \leq \gamma & \\ & & \hat{g} = 1. \\ & \textit{else} & \\ & & \hat{g} = \frac{\sigma_x^2 - \gamma}{\sigma_x^2 + \sigma_w^2 - 2\gamma}. \end{array} \tag{8}$$

where

$$\gamma = (b - \sigma_x \sigma_w) F(b) + \sigma_x \sigma_w. \tag{9}$$

III. Experimental Results

The first estimator discussed in this paper is applied to test the correlation between the signal and the quantization noise generated by the H.264 codec and to filter reconstructed intra-coded frames. The initial set of noisy reconstructed intra frames were generated using the H.264 codec (JM94). Frames from each video sequence were coded as intra-frames and reconstructed without use of H.264 post-filtering. The 4x4 transform option is used in H.264.

Three postfilters were applied to these frames. They include two overcomplete domain filters—using the minimax and the first BU-EST estimator, and the spatial-domain H.264 postfilter. The first two filters use a block DCT as a shifted transform at 4x4 locations around a block position in a frame to generate the overcomplete transform representation. Also, a filtered image sample is obtained by the standard method of averaging its overcomplete estimates. A linear, division-less approximation of the BU-EST in Eq. (5) is used for current experiments.

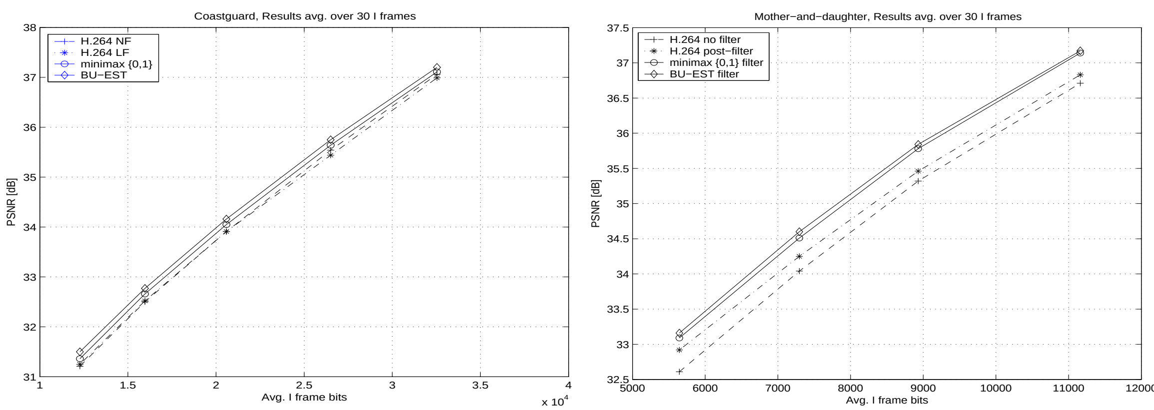

Fig. 2. Sample rate-distortion curves.

Fig. 2 shows preliminary results in terms of sample RD curves obtained by application of filtering to frames from two video sequences having different characteristics. The BU-EST postfilter gives a gain of about 0.25 dB over the H.264 postfilter in the Coastguard sequence (in fact the H.264 LF over-filters at higher PSNRs here), and up to 0.35 dB gain for the Mother-and-daughter sequence. Relative to the minimax postfilter, it gives a gain of up to 0.15 dB for Coastguard (which has significant high frequency content), and up to 0.1 dB for the second sequence. In the first case, the BU-EST postfilter essentially doubles the gains of the minimax postfilter over the H.264 filter.

IV. Summary

In this paper, we present the determination of a class of bounded-uncertainty estimators. These estimators are parameterized by the known (estimated) statistics of the problem as well as by bounds placed on the unknown statistics. The derivation is made under the realistic assumption of correlated signal and noise. The design presented affords a more general estimator formulation. It can also enable performance gains when the statistics of unknown variables are in reality bounded below the worst-case limit which is assumed by a classic minimax estimator.

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